INTERACTIVE CONSTITUTION OF MATHEMATICS BY TEACHER EDUCATION STUDENTS

Bob Perry, Noel Geoghegan, Peter Howe, Kay Owens
Faculty of Education
University of Western Sydney, Macarthur

Abstract

The New South Wales Minister for Education has decreed that all teachers wishing to gain initial employment from 1996 in New South Wales government primary schools need to have successfully studied the equivalent of 2 units of mathematics at the Higher School Certificate level. In response, the authors are investigating an alternative approach for teaching such material to primary teacher education students who have not reached this standard of mathematics. The approach was trialled in 1993 and, in 1994, the approach is being used with further groups.

In this paper, the theoretical foundations for the approach are discussed, along with preliminary results concerning critical changes in the students' development of their mathematical ideas and in their beliefs, attitudes, and values pertaining to mathematics.

Introduction

The NSW Minister of Education announced that, by 1996, beginning teachers in NSW government schools would need to have completed successfully the equivalent of two units of mathematics at Higher School Certificate level. At the University of Western Sydney, Macarthur, many teacher education students enter their course without this level of mathematics. The Faculty of Education now offers an elective subject, *Mathematics for K-6 Teachers*, to assist these students.

There seems to be little justification in repeating the same approaches to mathematics learning as might be commonly experienced in high schools and an alternative approach has been developed. The key constructs around which this approach is built are an *experiential learning cycle* adapted from Jones & Pfeiffer (1975) which uses principles of *cooperative learning* and the *problem-centred approach* of the Purdue Mathematics Project (Cobb, Wood & Yackel, 1991; Wood, Cobb & Yackel, 1992; Wright, 1992). Details of the approach have been reported previously (Perry, in press).

The project reported in this paper was supported by the Committee for the Advancement of University Teaching and the University of Western Sydney, Macarthur. The opinions expressed do not necessarily reflect the views of these funding agencies.

This paper provides a preliminary discussion of results arising from one class using the approach in 1994. This class has had one of the authors as its teacher.

Background to the Approach

Cooperative small group learning.

Owens (1993) has reviewed extensively the use of cooperative learning approaches in mathematics education and, based on this review, has suggested that *"many authors have advocated the use of cooperative groups"* (p. 24) and that *"Advocates of cooperative small group learning suggest that there are a number of important outcomes of small group learning. These include higher achievement, creative productivity, intrinsic motivation, positive self-esteem, positive social development, divergent thinking, effective problem solving, and development of thinking skills at higher cognitive levels."* (loc cit). Further, she provides evidence that *"small groups provide opportunities for collaborating dialogue which encourages active cognitive involvement and the resolution of conflicting opinions."* (loc cit). Griffin (1993) defines cooperative learning and its benefits in the following way:

> *The cooperative learning approach involves a learning environment where students can achieve their own individual goals only by working in combination with others. It contrasts with competitive and individual learning, generally found in traditional education settings. Fundamental to cooperative learning is the active role created for everyone by forming small aggregates of students as interacting units, in contrast to teacher-student individual interactions which disregard the student's relationship to their peers.* (p. 321)

In the approach taken with the subject *Mathematics for K-6 Teachers*, cooperative small group learning techniques have been adopted.

Experiential learning cycle.

The cycle used in *Mathematics for K-6 Teachers* consists of four stations: *Experiencing*, *Discussing*, *Generalising*, and *Applying*. Worksheets have been prepared which utilise this cycle to encourage small groups of students - either pairs or threesomes - to become actively involved with mathematical problems; to talk about their solution attempts, both in their groups and with other groups; to share their attempts with the whole class in an effort to generalise solutions to the stage where the whole class is prepared to accept a solution or solutions as taken-as-shared (Cobb. Perlwitz & Underwood, 1992; Wood, Cobb, Yackel, 1991) and, then, apply these to further problems.

Social norms of the class.

A key feature of the approach taken in *Mathematics for K-6 Teachers* is the interactive construction of a set of social norms within the class. The following norms have been developed.

1. Activities will consist of problems for the students. That is, it is assumed that the students may not be able to obtain solutions or even know where to start, immediately.

2. When working in small groups, students are expected to cooperatively develop solutions to the activities and to reach consensus on these solutions. The teacher is expected to circulate among the groups, observing their interactions and encouraging their problem solving attempts.

3. Students are expected, as a small group, to explain and defend their solutions or attempts at solutions to the whole class. Other students are expected to indicate their agreement or disagreement and to encourage alternative solutions.

4. The whole class is expected to see itself as a community of validators and is expected to work towards a solution or solutions which can be taken-as-shared. It is not the teacher's role to validate solutions.

These norms have been established within the *Mathematics for K-6 Teachers* class, although it has taken some time for the class to agree on the fourth one. Nevertheless, the teacher has been encouraged by the following:

> *... when a student has struggled to find an answer to a given problem, it is not only boorish but also counterproductive to dismiss it as 'wrong', even if the teacher then shows the 'right' way of proceeding. Such disregard for an effort made inevitably demolishes the student's motivation. Instead, a wiser teacher will ask the student how he or she came to the particular answer.* (von Glasersfeld, 1992, p. 8)

There have been many instances in *Mathematics for K-6 Teachers* where 'wrong' answers have led, through sharing and discussion to refinements of approach and further understanding.

Methodology

The exploratory nature of the learning approach used in *Mathematics for K-6 Teachers* requires a variety of data gathering procedures. As well, the adoption of a constructivist paradigm in the teaching / learning approach has resulted, to some extent, in a similar approach being required in the research aspects of the project. Consequently, while the data gathering procedures are well established, the analytical procedures utilised to consider the data are still in their formative stages. The team continues to refine these.

Four major data gathering techniques are being used.

1. *Videorecording.*

Each of the lessons given in the 1994 class of *Mathematics for K-6 Teachers* has been videorecorded using two cameras and a sophisticated sound arrangement. One member of our author team has taught all the classes while the other members have directed the recording and observed the classes.

2. *Generalisers.*

Each student is expected to record in a journal - called their Generaliser - their reactions to the course, attempts at solutions to the activities and any other feelings or concerns they may have.

3. *Assignments.*

As part of the subject, students are required to undertake assignments in pairs and an individual final investigation.

4. *Surveys.*

At the beginning of the semester and at the end, each student was asked to complete four short instruments designed to measure their attitude to mathematics and their beliefs about mathematics, mathematics learning and mathematics teaching. These instruments are available in Perry, in press.

5. *Reflective interviews.*

It is intended that, at the end of the semester, students will be shown a number of excerpts from the videorecordings of the classes and be asked to reflect on these. As well, oral questions will be asked concerning the students' perceptions of the approach taken in the classes.

Through the analysis of this large collection of data, it is expected that categories of critical change in the development of the mathematical ideas of students will be identified. At the time of writing this paper, videorecorded and Generaliser data were available for only the first five weeks of classes and only one assignment had been completed. Survey data on beliefs and attitudes taken before classes commenced were available but have not been considered in this paper. Nonetheless, preliminary categories for critical change can be identified.

Preliminary Results

Categories of individual student's activity, within the small groups or the whole class, which represent potential critical-change points in the mathematics learning of this student have been identified tentatively by the authors. They are presented here, with examples of dialogue from the *Mathematics for K-6 Teachers* class, to indicate the authors' current analysis of the data and to provide the basis for discussion and feedback from colleagues. There is a great deal more analysis to be completed as the data pool becomes larger. In each excerpt, students are indicated by S1, S2, ... but, for example, student S1 in one excerpt is not necessarily the same as student S1 in another excerpt.

In the cooperative, problem-centred approach to mathematics learning advocated in this project, potential critical-change points in students' learning may occur in the following circumstances.

1. The student realises that there is empathy within the group for the struggle being undertaken

17/3:1: 11.00 - 11.10 (date:tape number: hours.minutes.seconds (start) - hours.minutes.seconds (end)

S1: Both of you show us how to do it cause I don't know what either of you are on about.

S2: Good on you mate, exactly, I'm on your side.

S1: I think most people would be.

17/3:1: 21.15 - 21.40

S1: I still don't know what the hell you are on about. I don't want to be

S2: Second the motion. ...

S1: Cool. I just don't know what's happening here.

S3: Thank you for saying that.

S1: Oh, they just started going on about factors and I'm saying 'Yeah'

Through such empathy, the learners bond together and become willing to share their findings.

2. The student realises that the problem situation can be separated from the mathematics which might be needed to solve it and which she / he was beginning to construct.

17/3:1: 24.35 - 25.14

S1: So 76 has two factors - 38 and 2. You know you said the 38th door, every 38, right?

S2: Yeah

S1: So you turn door 38 one way, right, so it will have obviously one, and then the next from 38 is 76 will have 2 and 38. So that's how the factors work in, so it's got two multiples.

S2: Doesn't every door, every door gets turned at least once?

S1: Yes, depending on how many multiples go into it, depending on how many multiples go into it.

S2: Depending on how many multiples go into that number

S1: Into that number

S2: Depends on how many times the door gets turned. Yeah, yeah, I'm with you.

This excerpt also indicates the assistance a story or picture associated with the problem can be.

3. The student realises, through interaction with other members of the learning community, that a certain orientation or strategy may prove fruitful.

17/3:1: 34.00 - 34.45

S1: Why has 2 got two dots?

S2: Factors of 2.

S1: Why has 3 got two dots?

S2: 1 and 3 are the only factors of 3 and the factors of 4 are

S1: 2 and 2 and 1 and 4, I get you.

S2: 1 and 4 and 2 and 2 but when its double, you cross the second one out, so there's three factors of four.

S3: No, why do you cut it out?

S2: Cause it's doubled up you only have one.

S3: OK, OK.

S2: So that's how he just used dots instead of numbers.

4. The student realises that there is certain mathematical knowledge, to which she / he has access already, which can be applied to the problem.

24/3:1: 16.10 - 16.28

T: Can you tell me what value coin number 72 will have?

S1: Yeah, it's working with factors again, isn't it? ... Is it working with, well, you're not going to answer me but it will be working with factors again.

Of course, this is also an interesting comment on the construction of the class norms.

17/3:1: 21.50 - 22.20

S1: I'm in the same boat as you over here and I've just been enlightened, right. Probably, right, just my boat's overturned but I think instead of I think instead of going through and saying like you've got to open every third door, so instead of going through and going walking down the corridor and going 1, 2, 3 open, 1, 2, 3 open, 1, 2, 3 open, the easiest way of doing it so that you don't have to physically go through and do that is to find the factors of the number of the one you're supposed to open and that's what the factor thing is.

5. Through the processes of collaborative validation of attempted solutions, the student realises that a solution is viable and will be taken-as-shared by the learning community.

Such validation occurs in the *Mathematics for K-6 Teachers* class in a number of ways. There is a stage when a student's attempted solution appears possible in the light of discussions held within the group and this provides the confidence for the student to declare that it 'makes sense'. However, even when this is the case, students often need to 'do it for themselves'. Beyond this, there is a stage when the class seems to agree on a solution presented - the stage of the solution being taken-as-shared by the group. There are still a number of students who seek further validation of their solutions through the teacher and can get quite upset when he refuses to provide such validation.

24/3:2: 0.55 - 2.25

T: So we know how to do this, but we've still got two different answers.

S1: I prefer that one (pointing) ...

S2: We can just average them.

T: You can't just average them. ...

S3: Did everyone get different answers? ...

T: How are we going to decide (which answer is correct)?

S2: We're not going to bother.

S4: (to teacher) We're going to get you up there and you're going to do it for us.

S2: It doesn't matter.

T: Yeah, it does, it does matter.

S2: It doesn't really matter.

T: OK, then we won't worry about it.

S3: Why don't we do it as a group?

S5: No, No!

S2: There's always somebody.

The class continued for a further 30 minutes until an agreed solution was reached.

17/3:1: 19.45 - 20.15

S1: I don't think that if I was going to do it even now, I think I would have to go back and do 1 to 100 and do it step by step, even though, but I can see

S2: That's exactly what I did

S1: You wrote 1 to 100 and

S3: It was only once we had finished it that we picked out it was only the square numbers.

S4: That's what I would have to do. There's no way that I could sort of try to get the factors and try to get the pattern from 1 to 100 and try to figure it out.

17/3:1: 8.20

S1: Hopefully with these two, we might be able to get an answer. Their arguing, though.

Discussion

Even though the above results are quite tentative, they show some of the values of the approach being undertaken in the subject *Mathematics for K-6 Teachers*. The cooperative, problem-centred approach has facilitated the mathematics learning of many of the students in the class and has developed in them a confidence in their own abilities to, at least, get started on mathematical problems. The interactive constitution of the social norms within the learning community has meant that the students feel comfortable with the approach and what it is attempting to do.

The tentative identification of categories of activity potentially leading to critical change in the students' mathematics learning has provided a platform from which to study relationships between the activities undertaken within the cooperative, problem-centred class and these points of change. The initial indications are that the approach will help student teachers interactively constitute their mathematics, not only to meet an immediate employment requirement but also to model an approach which will have viability in their own teaching.

References

Cobb, P., Perlwitz, M. & Underwood, D. (1992). *Constructivism and activity theory: A consideration of their similarities and differences as they relate to mathematics education.* Paper presented at the Seventh International Congress on Mathematical Education, Quebec City.

Cobb, P., Wood, T. & Yackel, E. (1991). A constructivist approach to second grade mathematics. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education.* (pp. 157-176). Dordrecht: Kluwer.

Griffin, J. (1993). An integrated teaching model designed to increase the quality of science learning by primary teacher education students. In J. Bain, E. Lietzow & B. Ross (Eds.). *Promoting teaching in higher education.* (pp. 319-336). Brisbane: Griffith University.

Jones, J. & Pfeiffer, J. (Eds.) (1975). *The 1975 annual handbook for group facilitators.* San Diego: University Associates.

Owens, K. D. (1993). *Spatial thinking processes employed by primary school students engaged in mathematical problem solving.* Submitted doctoral thesis, Deakin University.

Perry, B. (in press). Primary teacher education students constructing their mathematics. In M. Carss (Ed.) *Proceedings of the eleventh biennial conference of the mathematics education lecturers association.* Brisbane: University of Queensland.

von Glasersfeld, E. (1992). *Aspects of radical constructivism and its educational implications.* Paper presented at the Seventh International Congress on Mathematical Education, Quebec City.

Wood, T., Cobb, P. & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, **28**(3), 587-616.

Wood, T., Cobb, P. & Yackel, E. (1992). Change in learning mathematics: change in teaching mathematics. In H. Marshal (Ed.), *Redefining student learning: Roots of educational change.* (pp. 177-205). Norwood, NJ: Ablex.

Wright, R. (1992). *Intervention in young children's arithmetical learning: The development of a research-based mathematics recovery program.* Paper presented to the Annual Conference of the Australian Association for Research in Education, Geelong, VIC.